

Mathematical Approach to Various Factors Affecting Human Health by Using Fuzzy Logic

Mukesh K. Sharma

Department of Mathematics,
R.S.S. (PG), Pilkhuwa, (Hapur)

ABSTRACT:

The aim of the present work is to characterize the failure rate of the basic factors affecting human health. In present paper basic factors are treated to be a possibility function instead of a crisp number. The possibility failure rate effectively captures the vagueness nature as well as the behavior of the system. Thus this approach seems to be more practical in comparison to conventional approach. The generalization of Boolean operators AND & OR also brings our approach much closer to the actual behavior of system. The method, used to get best possible fuzzy number out of more than one fuzzy number for any single factors, also becomes a very effective tool to meet the fuzziness present during the estimation of possibility function (fuzzy numbers) for the basic factors. The trapezoidal fuzzy numbers are chosen due to their efficiency and simplicity. Further α -cut ranking method for all the basic factors can also be used and be more effective method for ranking the basic factors.

KEYWORDS: Fuzzy Sets, Membership function, Fuzzy Numbers, Triangular Fuzzy Numbers, Trapezoidal Fuzzy Numbers and Fuzzy Operators.

1 INTRODUCTION:

Our physical health is influenced and determined by what we eat (AHAR), the environment we live in (VIHAR) and the way we think, perceive and act (VICHAR). We may have control over these factors only to some extent but that is enough to have a positive outcome. The nutrition or the food we eat remains the most important influencing component of this triad. The food we eat gives us the energy and ability to grow physically when we are young and maintain the body after physical growth ceases. The energy sources are carbohydrates and fats while growth and maintenance is because of proteins. Besides these, we need minerals and vitamins, which are essential to carry normal body functions. Water is another vital input. Can we aspire the healthy aging? It is possible with a combination of discipline (SANYAM), balance (SANTULAN) and commitment (SAMARPAN) It may be difficult, but certainly not impossible, if one eats whole grain foods, drinks plenty of water and properly manages work schedules to minimize work stress. To make up for the shortfall (the nutrition gap) of vitamins and minerals, we have to take natural supplements of best quality on daily basis. This is the concept of cellular nutrition i.e. strengthening the most basic unit of our body, the cell.

All of us need supplements because we are exposed to same risks. These vital elements not only pfactor the diseases, but help in recovery from illness by strengthening the immune system. In fact all systems of medicines advise to consume plenty of fresh fruits and vegetables which are the main sources of vitamins and minerals, to keep healthy and pfactor the chronic diseases. World Health Organization (WHO) recommends to have approximately 400 gms. fruits and vegetables daily. Certainly it is a difficult advice to be followed due to practical reasons, but the requirement can be met by daily intake of good quality natural multivitamin supplement. This supplements only one vital aspect of healthy living. A regular exercise program, adequate rest, positive outlook, wholesome meals, quitting the smoking, moderation in alcohol intake, stress management are other inputs for a good overall health.

2 FACTORS HAVING POSITIVE EFFECTS ON HUMAN HEALTH:

2.1 FOOD:

The body is like a machine and to run it, fuel is needed. The food that we eat supplies it. The main functions of the food are as follows

- 1.To supply energy i.e. Energy giving food.
- 2.To supply material for tissue building and growth i.e. body building food.
- 3.To help replenish worn out cells and tissues i.e. strength giving food.

2.2 CARBOHYDRATES:

Carbohydrates are principle source of energy. These are compounds of carbon, hydrogen and oxygen. Generally hydrogen and oxygen are in the proportion to form water hence the name carbohydrate. These are foods that are simple sugars or complexes (Polymers) of sugars (starch).

2.3 FATS:

Fats are the concentrated source of energy usually called reserve energy. These are greasy substances, which are insoluble in water but soluble in certain organic solvents like ether, alcohol, benzene etc. A fat has following valuable roles to play:

1. Essential fatty acids boost the immune system and ward off disease.
2. Fats helps in release of sugar into our bloodstream, important for hypoglycemic and diabetics.
3. Fat is needed for the absorption of vitamins A, E, D, K and beta - carotene.
4. Fat is needed to form all cell membranes.

2.4 NUTRITION:

Principal components of nutrition are proteins, carbohydrates and fats, vitamins, minerals, trace elements, electrolytes and water. Nutritional elements may be divided in three or more groups.

1. Providers of energy and heat - carbohydrates and fats
2. (Body) tissue builders - Proteins.
3. Regulators of vital processes - Vitamins, minerals, Electrolytes, trace elements.
4. Medium and transport of nutrients - water.

2.5 PROTEINS:

Without proteins no life appears to be possible. Proteins are nitrogenous compounds and form principal constituents of protoplasm of plants and animal tissues. Amino acids are the basic units. Proteins contain carbon, hydrogen, oxygen and nitrogen. Proteins consist of chains of amino acids joined to each other by peptide linkage. Two amino acids form dipeptide, three form tripeptide and so on.

2.6 MICRONUTRIENTS:

Vitamins and minerals promote essential biochemical reaction in the cell of the body. Multivitamins help us in the cardiovascular system, the nervous system, the circulatory system, tissue and cell regeneration and muscular response. Vitamins and minerals together are called micronutrients.

Metabolism: Metabolism is the process where by chemical changes, both building up and breaking down, enable the functions of nutrients to be affected. Metabolism is divided under two main heads.

- a) Anabolism or constructive process or synthesis e.g. Amino acids into proteins.
- b) Catabolism implies breakdown of large molecules into smaller size. Glycogen to glucose to carbon dioxide and water

Everyone has different metabolisms. So it is difficult to say which supplements would be most important for us. For some people zinc is important, but it must be balanced with copper to suit our individual needs. Beta -carotene is a very important antioxidant.

3 FACTORS HAVING NEGATIVE EFFECT ON HUMAN HEALTH:

For normal and healthy living, a conducive environment is required by all the living beings including humans, plants, micro-organisms and the wild life. All the biological and non-biological things surrounding an organism are included in environment. The favorable unpolluted environment has a specific composition. In this way the environment gets polluted. Environmental pollution can, therefore, be defined as any undesirable change in the physical, chemical or biological characteristics of any component of the environment (air, water, soil), which can cause harmful effect on the survival and health of the living beings. Pollution may be of following types:

3.1 AIR POLLUTION:

It is an atmospheric condition in which certain harmful substances (including the normal constituents in excess) are present in air. These substances cause air-pollution that has an undesirable effect on the health. These substances include gases, particulate matter, radioactive substances etc.

3.2 WATER POLLUTION:

Water pollution can be defined as alternation in physical, chemical or biological characteristic of water making it unsuitable for designated use in its natural state. Water is an essential commodity for survival.

3.3 NOISE POLLUTION:

We have various types of sounds everyday. Sound is mechanical energy from vibrating sources. A type of sound may be pleasant to someone and at the same time unpleasant to others. The unpleasant and unwanted sound is called noise pollution.

3.4 THERMAL POLLUTION:

Thermal pollution can be defined as presence of waste heat in water, which can cause undesirable changes in the natural environment. Thermal power plants, nuclear power plants, refineries, steel mills etc. are the major sources of thermal pollution. Power plants utilize only 1/3 of the energy provided by fossil fuels for their operations. Remaining 2/3 is generally lost in the form of heat to the water used for cooling. Excess of heat reaching in water causes thermal pollution of water.

3.5 NUCLEAR HAZARDS:

Radioactive substances are present in nature. Various sources of radioactive substances can be grouped into natural and anthropogenic (man made) sources. Many scientists are of the view that due to the body's ability to repair some of the damages, the adverse effects of radiations are observed only beyond a threshold level.

3.6 GLOBAL WARNING:

All living beings inhale a considerable quantity of air. An average human being breath about 22,000 times a day and inhales about 16 kg. of oxygen. The pollution of air, therefore, may have a profound influence on living organisms. Unfortunately, a large quantity of gases, particulate materials, fumes, vapors and smoke is discharged daily into the atmosphere.

3.7 PESTICIDES:

Pesticides are group of large number of chemicals which are used to suppress or eliminate undesirable organisms. Most of these chemicals are poisonous substances capable of damaging one type of organism drastically while causing none or only nominal damage to the desired one, even if the two are in a close association. Following are three major groups:

- a) Insecticides b) Herbicides c) Fungicides

4 FUZZY NUMBERS AND ARITHMETIC OPERATIONS:

(a) FUZZY NUMBERS:

A fuzzy set A defined on a universal set R must possess the following properties to qualify as a fuzzy number.

- (a) $\mu_A(x) = 0$ for all $x \in (-\infty, c] \cup [d, \infty)$
- (b) $\mu_A(x)$ is strictly increasing on $[c, a]$ and strictly decreasing on $[b, d]$
- (c) $\mu_A(x) = 1$ for all $x \in [a, b]$

(i) TRIANGULAR FUZZY NUMBERS:

A fuzzy number A is termed as triangular fuzzy number if the membership function of fuzzy number A is defined by the following expression:

$$\mu_A(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & \text{if } a_1 \leq x \leq a_2 \\ 0 & \text{if } x \leq a_1 \text{ or } x \geq a_3 \\ \frac{a_3-x}{a_3-a_2} & \text{if } a_2 \leq x \leq a_3 \end{cases}$$

The triangular fuzzy number may be denoted by an ordered triad i.e. if A is fuzzy number given by above expression then it can be given by the triad $A=(a_1, a_2, a_3)$. The membership function of this number may be depicted.

(ii) TRAPEZOIDAL FUZZY NUMBERS:

If the membership function of a fuzzy number A is expressed in the following form then A is said to be a trapezoidal fuzzy number.

$$\mu_A(x) = \begin{cases} 0 & \text{if } x \leq a_1 \text{ or } x \geq a_4 \\ \frac{x-a_1}{a_2-a_1} & \text{if } a_1 \leq x \leq a_2 \\ 1 & \text{if } a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3} & \text{if } a_3 \leq x \leq a_4 \end{cases}$$

where $x, a_1, a_2, a_3, a_4 \in R$

(b) OPERATIONS ON TRIANGULAR AND TRAPEZOIDAL FUZZY NUMBERS:

Let $A=(a_1, a_2, a_3)$ and $B=(b_1, b_2, b_3)$ be two triangular fuzzy numbers. Then the fuzzy addition of triangular fuzzy number A and B is defined as

$$\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$$

Therefore addition of two triangular fuzzy numbers is again a triangular fuzzy number.

Similarly subtraction two triangular fuzzy numbers is also a triangular fuzzy number that can be given by the following expressions: -

$$A-B = (a_1-b_1, a_2-b_2, a_3-b_3)$$

Multiplication of two triangular fuzzy numbers A and B need not to be a triangular fuzzy number. The membership function for multiplication of two fuzzy numbers $A=(a_1, a_2, a_3)$ $B=(b_1, b_2, b_3)$ may be given by the following expression.

$$\mu_{\tilde{A} * \tilde{B}}(x) = \begin{cases} -D_1 + [D_1^2 + (x-P)/T_1]^{1/2} & P \leq x \leq Q \\ -D_1 - [D_1^2 + (x-R)/U_1]^{1/2} & Q \leq x \leq R \\ 0 & \text{otherwise} \end{cases}$$

Where $T_1 = (a_2-a_1)(b_2-b_1)$, $T_2 = a_1(a_2-a_1) + b_2(b_2-b_1)$,

$$U_1 = (a_2-a_1)(b_2-b_1), U_2 = b_3(a_2-a_1) + a_3(b_2-b_1), D_1 = \frac{T_2}{2T_1}, D_2 = -\frac{U_2}{2U_1}, P = a_1b_1, Q = a_2b_2, R = a_3b_3$$

From the above expression it is evident that $A*B$ is not a triangular fuzzy number. However to make computations simpler, we avoid the second or higher degree terms and assume the product to be a triangular fuzzy number (P, Q, R) , that is, (a_1b_1, a_2b_2, a_3b_3) .

The algebraic operations on trapezoidal fuzzy numbers are also defined in a similar manner. Let $A=(a_1, a_2, a_3, a_4)$ and $B=(b_1, b_2, b_3, b_4)$ be two trapezoidal fuzzy numbers. Then the addition and subtraction of these two trapezoidal fuzzy numbers is represented by two trapezoidal fuzzy numbers $(a_1+b_1, a_2+b_2, a_3+b_3, a_4+b_4)$ and $(a_1-b_1, a_2-b_2, a_3-b_3, a_4-b_4)$ respectively. Also the product of these two trapezoidal fuzzy numbers needs not to be trapezoidal. In a manner similar to the triangular fuzzy numbers, product of two trapezoidal fuzzy numbers A and B is approximated as a trapezoidal fuzzy numbers given by $(a_1b_1, a_2b_2, a_3b_3, a_4b_4)$

5 FUZZY OPERATORS:

Using algebraic operations on fuzzy numbers (triangular or trapezoidal), we can obtain fuzzy operators corresponding to Boolean operators AND, OR etc. This may be defined by the following expressions.

Let $\tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_n$ are the possibility functions of the basic factors. Then fuzzy AND operator can be defined as below and denoted by ANF

$\tilde{p}_y = ANF(\tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_n) = \prod_{i=1}^n \tilde{p}_i$, where \prod denotes the fuzzy multiplication and p_y be the possibility of resulting factor.

Let \tilde{p}_i 's are represented by triangular fuzzy numbers i.e.

$$\tilde{p}_i = (a_{i1}, a_{i2}, a_{i3}) \quad i=1, 2, \dots, n$$

Then $\tilde{p}_y = ANF(\tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_n) = (a_{y1}, a_{y2}, a_{y3})$,

where a_{y1}, a_{y2}, a_{y3} denotes the following terms

$$a_{y1} = \prod_{i=1}^n a_{i1}, \quad a_{y2} = \prod_{i=1}^n a_{i2}, \quad a_{y3} = \prod_{i=1}^n a_{i3}$$

If p_i 's are the trapezoidal fuzzy numbers i.e. $p_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4})$. Then

$$\tilde{p}_y = ANF(\tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_n) = (a_{y1}, a_{y2}, a_{y3}, a_{y4}),$$

where $a_{y1} = \prod_{i=1}^n a_{i1}, \quad a_{y2} = \prod_{i=1}^n a_{i2}, \quad a_{y3} = \prod_{i=1}^n a_{i3}, \quad a_{y4} = \prod_{i=1}^n a_{i4}$

Now we define the fuzzy form of OR operator i.e. $ORF(\tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_n)$ as

$$\tilde{p}_y = ORF(\tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_n) = 1 - \prod_{i=1}^n (1 - \tilde{p}_i)$$

If $\tilde{p}_1, \tilde{p}_2, \dots, \tilde{p}_n$ are triangular fuzzy numbers, then $1 - \tilde{p}_1, 1 - \tilde{p}_2, \dots, 1 - \tilde{p}_n$, are also triangular i.e. if $\tilde{p}_1 = (a_{i1}, a_{i2}, a_{i3}), i=1, 2, \dots, n$, then

$$\begin{aligned} \tilde{p}_y &= (a_{y1}, a_{y2}, a_{y3}) = 1 - \prod_{i=1}^n (1 - (a_{i1}, a_{i2}, a_{i3})) \\ &= 1 - \prod_{i=1}^n (1 - a_{i3}, 1 - a_{i2}, 1 - a_{i1}) = 1 - \left(\prod_{i=1}^n (1 - a_{i3}), \prod_{i=1}^n (1 - a_{i2}), \prod_{i=1}^n (1 - a_{i1}) \right) \\ &= (1 - \prod_{i=1}^n (1 - a_{i1}), 1 - \prod_{i=1}^n (1 - a_{i2}), 1 - \prod_{i=1}^n (1 - a_{i3}),) \end{aligned}$$

Similarly if p_i 's are trapezoidal fuzzy numbers i.e. $p_i = (a_{i1}, a_{i2}, a_{i3}, a_{i4})$. Then

$$ORF(p_1, p_2, \dots, p_n) = (1 - \prod_{i=1}^n (1 - a_{i1}), 1 - \prod_{i=1}^n (1 - a_{i2}), 1 - \prod_{i=1}^n (1 - a_{i3}), 1 - \prod_{i=1}^n (1 - a_{i4}))$$

6 APPROXIMATIONS OF FUZZY NUMBERS FOR BASIC FACTORS:

Suppose data about the occurrence of the basic factors are provided to n experts and they are asked to assign a possibility function (fuzzy number) to it. Let us assume their answers are in the form of triangular fuzzy numbers such as $A_i=(a_i-c_i, a_i, a_i+c_i) \ i=1, 2 \dots n$.

Now our task is to find out a triangular fuzzy numbers that tunes with the judgment of all experts. Let $B=(b-d, b, b+d)$ be the number, which fits with experts' decision. All A_i 's are used to determine the parameters b and d . For this we find $B-A^i$, which is again a triangular fuzzy number. Smaller triangular fuzzy numbers will result in the better approximation for B . The height of the triangle $B-A^i$ cannot be reduced, since it must always be one. Therefore our measure depends on the length of base line of the triangle. For this we suppose

$$S = \sum [2(d-c_i)]^2$$

Then S will achieve its minimum if $d = \frac{1}{n} \sum_{i=1}^n c_i$.

Further for the determination the parameter b , we suppose

$$D = \max_{1 \leq i \leq n} |b - a_i|$$

$$\min_{1 \leq i \leq n} a_i + \max_{1 \leq i \leq n} a_i$$

Then D will be minimum for $b = \frac{\min_{1 \leq i \leq n} a_i + \max_{1 \leq i \leq n} a_i}{2}$

This approach may also be applied to deal with trapezoidal fuzzy numbers. Let $A_i=(a_i-c_i, a_i, a_i', a_i'+c_i) \ i=1,2 \dots n$, are n trapezoidal fuzzy numbers assigned to the failure possibility of a basic factor. Using a similar approach, a trapezoidal fuzzy number $B=(b-d, b, b', b'+d)$ will be the fuzzy number that fits-in the best with all experts' decision where the parameters b, b' and d can be given the following expressions:

$$d = \frac{1}{n} \sum_{i=1}^n c_i,$$

$$b = \frac{\min_{1 \leq i \leq n} a_i' + \max_{1 \leq i \leq n} a_i'}{2} \quad \text{and} \quad b' = \frac{\min_{1 \leq i \leq n} a_i + \max_{1 \leq i \leq n} a_i}{2}$$

7 POSSIBILITY ANALYSES:

The undesired factors are graphically depicted by using fuzzy logic symbols such **OR, AND** and **NEG**. The basic factor (E_i) contains only mutually exclusive variables of tree structure system. The overall function of the system can be obtained by substituting all the independent variables (E_i) of the probabilities (P_{Ei}) and the transform of the logical operators OR and AND into algebraic addition and multiplication. The probability of NEG gate (E_i) is equivalent to $1- P_{Ei}$. Based on the main operations of the fuzzy number, the derivation of AND, OR and NEG operators for the human health can be obtained by substituting the crisp value and algebraic operation with fuzzy variables and fuzzy operation respectively. The AND and OR operator can be obtained as follows:-

AND operator functions:

$$P_{E_{TOP}}^{\alpha} = \prod_{i=1}^n P_{E_i}^{\alpha}$$

$$= AND(P_{E_1}^{\alpha}, P_{E_2}^{\alpha}, \dots, P_{E_n}^{\alpha})$$

$$= [a_{E_1}^{\alpha}, b_{E_1}^{\alpha}, c_{E_1}^{\alpha}] \otimes [a_{E_2}^{\alpha}, b_{E_2}^{\alpha}, c_{E_2}^{\alpha}] \otimes \dots \otimes [a_{E_n}^{\alpha}, b_{E_n}^{\alpha}, c_{E_n}^{\alpha}]$$

OR operator functions:

$$\begin{aligned}
 P_{E_{TOP}}^{\tilde{\alpha}} &= \tilde{1} - \prod_{i=1}^n (\tilde{1} - P_{E_i}^{\tilde{\alpha}}) \\
 &= [1, 1, 1] - ([1, 1, 1] - [a_{E_1}^{\tilde{\alpha}}, b_{E_1}^{\tilde{\alpha}}, c_{E_1}^{\tilde{\alpha}}]) \\
 &\quad ([1, 1, 1] - [a_{E_2}^{\tilde{\alpha}}, b_{E_2}^{\tilde{\alpha}}, c_{E_2}^{\tilde{\alpha}}]) \dots ([1, 1, 1] - [a_{E_n}^{\tilde{\alpha}}, b_{E_n}^{\tilde{\alpha}}, c_{E_n}^{\tilde{\alpha}}])
 \end{aligned}$$

8 POSSIBILITY WEIGHTED INDEX:

The fuzzy weighted index is a scheme that evaluates the contribution of different basic factors to the failure possibility of the top factor. Each basic factor is eliminated to obtain its weight in the fault tree. Let us consider a fault tree structure that consists of k basic factors. Suppose P_{TOP} is the failure possibility of the top factor (TOP) and $P_{TOP(n)}$ denotes the failure possibility of the factor (TOP) in which the failure possibility for the n th basic factor is eliminated. We may conclude $P_{TOP} > P_{TOP(k)}$ by knowing that the failure possibility of the top factor is an increasing function with respect to the failure possibility of any basic factor. Based on the property that the weight index $W (P_{TOP}, P_{TOP(k)})$, represents the distance between the two fuzzy number, P_{TOP} and $P_{TOP(k)}$, the difference will indicate the degree of improvement after eliminating the failure occurrence of the k^{th} basic factor E_k .

The distance $\delta(\tilde{M}, \tilde{N})$ of the two α -level ($\alpha \in [0, 1]$) triangular fuzzy numbers,

$$\tilde{M} = [a_m^\alpha, b_m^\alpha, c_m^\alpha] \quad \text{and} \quad \tilde{N} = [a_n^\alpha, b_n^\alpha, c_n^\alpha]$$

is defined as

$$\delta(\tilde{M}, \tilde{N}) = 0.5 \left\{ \max(|a_m^\alpha - a_n^\alpha|, |b_m^\alpha - b_n^\alpha|) + |c_m^\alpha - c_n^\alpha| \right\}$$

Geometrically, $\delta(\tilde{M}, \tilde{N})$ is the size of the trapezoidal area covered by the lower base segment $|a_m^\alpha - a_n^\alpha|$ or $|b_m^\alpha - b_n^\alpha|$ (larger part), the upper base $|c_m^\alpha - c_n^\alpha|$, and height unity.

The ranking of the fuzzy number ($P_{TOP(k)}$) will determine the fuzzy importance of the basic factors. There are various methods in fuzzy ranking and no particular fuzzy ranking method is perfect. Each method has its advantages and disadvantages. We shall work on α -cut based method for fuzzy ranking defined as following:

Let \tilde{M} and \tilde{N} be fuzzy number with α -cuts $\tilde{M}_\alpha = [m_\alpha^-, m_\alpha^+]$ and $\tilde{N} = [n_\alpha^-, n_\alpha^+]$. \tilde{M} is called smaller than \tilde{N} , denoted by $\tilde{M} \leq \tilde{N}$, if $m_\alpha^- \leq n_\alpha^-$ and $m_\alpha^+ \leq n_\alpha^+ \quad \forall \alpha \in [0, 1]$

9 FUZZY IMPORTANCE INDEX:

If we are able to analyze the importance of different basic factors, we can make a proper order of importance of these factors. On improving the reliability of the factor having greater importance, we can improve the reliability of the system. Pon and Yun [1] have obtained the fuzzy importance index by using composite method. Here we see that the fuzzy importance of any factor is always calculated in the form of fuzzy importance index (FII). This FII may be evaluated by ranking fuzzy number ($P_T - P_{Ti}$) for $i=1, 2, 3 \dots n$. Where P_T is the possibility of absolute occurrence of basic factor i and P_{Ti} denotes the possibility of occurrence of top factor in absence of basic factor i . Various methods may be used for ranking of fuzzy numbers. Here the method we have taken is less complicated and very significant. This method can be explained as follows.

If we like to rank a fuzzy numbers ($P_T - P_{Ti}$)'s for $i=1, 2, \dots, n$, then first of all we have to find out MAX ($P_T - P_{Ti}$), $i=1, 2, \dots, n$, where the MAX operator on fuzzy numbers can be defined in the following manner.

Let we have $A_1, A_2, A_3, \dots, A_n$ different fuzzy numbers. Then MAX of these fuzzy numbers is defined as

$$\text{MAX} (A_1, A_2, A_3, \dots, A_n)(z) = \sup \min [A_1(x_1), A_2(x_2), A_3(x_3), \dots, A_n(x_n)]$$

After having the MAX of given fuzzy numbers, we try to get the distance of all these fuzzy numbers from their MAX. Hamming distance formula is used to find out the distance between two fuzzy numbers. The Hamming distance between two fuzzy numbers A and B is defined as

$$d_H(A, B) = \int_{[0,1]} |A(x) - B(x)| dx$$

The distance of these fuzzy numbers $P_T - P_{T_i}$ for $i = 1, 2, \dots, n$ from their MAX decides the rank of fuzzy numbers $P_T - P_{T_i}$. Smaller the distance of fuzzy number $P_T - P_{T_i}$ from MAX ($P_T - P_{T_i}$), $i = 1, 2, \dots, n$. in comparison to distance of $P_T - P_{T_2}$ from MAX ($P_T - P_{T_i}$) implies that fuzzy number $P_T - P_{T_i}$ is greater than $P_T - P_{T_2}$.

This concludes that the fuzzy importance index (FII) may be defined in the form of distance of P_T from P_{T_i} i.e.

$$FII(i) = d(P_T, P_{T_i})$$

7. DISCUSSION AND INTERPRETATION:

The aim of the present work is to characterize the failure rate of the basic factor in fuzzy manner. In present paper basic factors are treated to be a possibility function instead of a crisp number. The possibility failure rate effectively captures the vagueness nature as well as the behavior of the system. Thus this approach seems to be more practical in comparison to conventional approach. The generalization of Boolean operators AND & OR also brings our approach much closer to the actual behavior of system. The method, used to get best possible fuzzy number out of more than one fuzzy number for any single factor, also becomes a very effective tool to meet the fuzziness present during the estimation of possibility function (fuzzy numbers) for the basic factors. The trapezoidal fuzzy numbers are chosen due to their efficiency and simplicity. Further the Fuzzy weighted index (FWI) is also obtained to provide valuable information for all the basic factors by using a very effective method of ranking of fuzzy numbers by using the α -cut ranking method.

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